HW. # 4

Homework problems are taken from textbook. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Evaluate the determinants.

1. (a) The following points are given in cylindrical coordinates; express each in rectangular coordinates and spherical coordinates: $(1,45^\circ,1)$, $(2,\frac{\pi}{2},-4)$, and $(3,\frac{\pi}{6},0)$

   (b) Change each of the following points from rectangular coordinates to spherical coordinates: $(2,1,-2)$, $(0,3,4)$, and $(-2\sqrt{3},-2,3)$

2. Describe the geometric meaning of the following mappings in cylindrical coordinates:

   (a) $(r,\theta,z)\to(r,\theta,-z)$

   (b) $(r,\theta,z)\to(r,\theta+\pi,-z)$

   (c) $(r,\theta,z)\to(-r,\theta-\frac{\pi}{4},z)$

3. Describe the geometric meaning of the following mappings in spherical coordinates:

   (a) $(\rho,\theta,\phi)\to(\rho,\theta+\pi,\phi)$

   (b) $(\rho,\theta,\phi)\to(\rho,\theta,\pi-\phi)$

   (c) $(\rho,\theta,\phi)\to(2\rho,\theta+\frac{\pi}{2},\phi)$

4. (a) Describe the surfaces r = constant, θ = constant, and z = constant in the cylindrical coordinate system.

   (b) Describe the surfaces ρ = constant, θ = constant, and φ = constant in the spherical coordinate system.

5. Two surfaces are described in spherical coordinates by the two equations $\rho = f(\theta,\phi)$ and $\rho=-2f(\theta,\phi)$, where $f(\theta,\phi)$ is a function of two variables. How is the second surface obtained geometrically from the first?

6. A circular membrane in space lies over the region $x^2+y^2\le a^2$. The maximum z component of points in the membrane is b. Assume that (x, y, z) is a point on the

membrane. Show that the corresponding point (r, θ, z) in cylindrical coordinates satisfies the conditions $0 \le r \le a$, $0 \le \theta \le 2\pi$, $|z| \le b$.

7. Calculate the dot product of **x** = (1, -1, 0, 2) and **y** = (1, 2, 3, 4)

8. In R^n show that

(a) $2\|x\|^2 + 2\|y\|^2 = \|x+y\|^2 + \|x-y\|^2$

(b) $4\langle x, y\rangle = \|x+y\|^2 - \|x-y\|^2$

9. In R^n show that $\|x\| \le \sum_{i=1}^{n} |x_i| \le \sqrt{n}\|x\|$ (Hint: Use triangle inequality and Cauchy – Schwarz)

10. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 6 & 7 & 1 \\ 0 & 4 & 2 \end{pmatrix}$, $C = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$, and $D = \begin{pmatrix} 2 & 5 \end{pmatrix}$. Evaluate the following or write that the expression is not defined.

(a) $AC + D^T$

(b) AB

(c) BA

(d) B^T

(e) $B^T C$